

GROUPS OF ORDERS 17-31

SUMMARY

order	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
# abelian	1	2	1	2	1	1	1	3	2	1	3	2	1	1	1
# other decomposable	0	1	0	0	0	0	0	6	0	0	0	0	0	2	0
# other indecomposable	0	2	0	3	1	1	0	6	0	1	2	2	0	1	0
TOTAL	1	5	1	5	2	2	1	15	2	2	5	4	1	4	1

$17 = \langle A^{17} \rangle$

$18 = 18 = \langle A^{18} \rangle = 9 \times 2$

$18.2 = 6 \times 2 = \langle A^6, B^2 \rangle$

$18.3 = \langle A^3, B^2, C^3, BA = A^{-1}B \rangle = D_6 \times 3$

C	1	2	3	4	5	6	7	8	9	
#	1	3	2	3	3	6	3	3	6	ℳ
χ_1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	-1	1	1	-1	1	1	-1	1
χ_3	2	-1	0	2	-1	0	2	-1	0	2
χ_4	1	1	1	ω	ω	ω	ω^2	ω^2	ω^2	3
χ_5	1	1	-1	ω	ω	$-\omega$	ω^2	ω^2	$-\omega^2$	4
χ_6	2	-1	0	2ω	$-\omega$	0	$2\omega^2$	$-\omega^2$	0	0
χ_7	1	1	1	ω^2	ω^2	ω^2	ω	ω	ω	3
χ_8	1	1	-1	ω^2	ω^2	$-\omega^2$	ω	ω	$-\omega$	4
χ_9	2	-1	0	$2\omega^2$	$-\omega^2$	0	2ω	$-\omega$	0	0
^	1	2	3	3	6	3	3	6	3	

Normal subgroups

	Classes	H	G/H	ℳ
1	1 + 2 + 4 + 5 + 7 + 8	3²	2	1
2	1 + 4 + 7	3	D₆	2 Z = Z₂ = ℳ
3	1 + 2 + 3	D₆	3	3
4	1 + 2	3	6	4 G'

Sylow subgroups: [2] × 3, [3²] **Maximal subgroups:** [6] × 3, [D₆], [3²]

$$18.4 = \langle A^3, B^3, C^2, CA = A^{-1}C, CB = B^{-1}C \rangle$$

CC	1	2	3	4	5	6
elts	1	A A ²	B B ²	AB A ² B ²	AB ² A ² B	A ^m B ⁿ C

C	1	2	3	4	5	6	
#	1	2	2	2	2	9	\mathcal{K}
χ_1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	-1	1
χ_3	2	2	-1	-1	-1	0	2
χ_4	2	-1	2	-1	-1	0	3
χ_5	2	-1	-1	2	-1	0	4
χ_6	2	-1	-1	-1	2	0	5
\wedge	1	3	3	3	3	2	

Normal subgroups

	Classes	H	G/H	\cap	
1	1 + 2 + 3 + 4 + 5	3^2	2	1	G'
2	1 + 2	3	D_6	2	
3	1 + 3	3	D_6	3	
4	1 + 4	3	D_6	4	
5	1 + 5	3	D_6	5	

$$Z = \mathcal{Z} = \Phi = [1]$$

Sylow subgroups: $[2] \times 9$, $[3^2]$ Maximal subgroups: $[3^2]$, $[D_6] \times 12$

$$18.5 = D_{18} = \langle A^9, B^2, BA = A^{-1}B \rangle$$

CC	1	2	3	4	5	6
elts	1	A ³ A ⁶	A A ⁸	A ² A ⁷	A ⁴ A ⁵	A ⁿ B

C	1	2	3	4	5	6	
#	1	2	2	2	2	9	\mathcal{K}
χ_1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	-1	1
χ_3	2	2	-1	-1	-1	0	2
χ_4	2	-1	c ₁	c ₂	c ₄	0	0
χ_5	2	-1	c ₂	c ₄	c ₁	0	0
χ_6	2	-1	c ₄	c ₁	c ₂	0	0
\wedge	1	3	9	9	9	2	

$c_k = 2\cos(2k\pi/9)$

Normal subgroups

	Classes	H	G/H	\cap	
1	1 + 2 + 3 + 4 + 5	9	2	1	G' Sylow subgroups: $[2] \times 9$, $[9]$
2	1 + 2	3	D_6	2	Φ Maximal subgroups: $[D_6] \times 3$, $[9]$

$$\mathcal{Z} = \Phi = [1]$$

$$19 = \langle A^{19} \rangle$$

$$20 = 5 \times 4 = \langle A^{20} \rangle$$

$$20.2 = 10 \times 2 = \langle A^{10}, B^2 \rangle$$

$$20.3 = D_{20} = \langle A^5, B^2, C^2, BA = A^{-1}B \rangle = D_{10} \times 2$$

	C	1	2	3	4	5	6	7	8	
#	1	2	2	5	1	2	2	2	5	\mathcal{K}
χ^1	1	1	1	1	1	1	1	1	1	G
χ^2	1	1	1	-1	1	1	1	1	-1	1
χ^3	2	c_1	c_2	0	2	c_1	c_2	c_2	0	2
χ^4	2	c_2	c_1	0	2	c_2	c_1	c_1	0	2
χ^5	1	1	1	1	-1	-1	-1	-1	-1	3
χ^6	1	1	1	-1	-1	-1	-1	-1	1	4
χ^7	2	c_1	c_2	0	-2	$-c_1$	$-c_2$	$-c_2$	0	0
χ^8	2	c_2	c_1	0	-2	$-c_2$	$-c_1$	$-c_1$	0	0
\wedge	1	5	5	2	2	10	10	10	2	

Normal subgroups

	Classes	H	G/H	\cap	
1	1 + 2 + 3 + 5 + 6 + 7	10	2	1	
2	1 + 5	2	D_{10}	2	Z
3	1 + 2 + 3 + 4	D_{10}	2	3	
4	1 + 2 + 3 + 8	D_{10}	2	4	
5	1 + 2 + 3	5	2^2	$1 \cap 2$	G'

$$\mathcal{K} = \Phi = [1]$$

Sylow subgroups: $[2^2] \times 5$, $[5]$ Maximal subgroups: $[D_{10}] \times 2$, $[10]$



$$20.4 = D_{5,4} = \langle A^5, B^4, BA = A^{-1}B \rangle$$

CC	1	2	3	4	5	6	7	8
elts	1	B ²	A	A ²	AB ²	A ² B ²	A ⁿ B	A ⁿ B ³
			A ⁴	A ³	A ⁴ B ²	A ³ B ²		

C	1	2	3	4	5	6	7	8	
#	1	1	2	2	2	2	5	5	\mathfrak{K}
χ_1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	-1	-1	1
χ_3	1	-1	1	1	-1	-1	i	-i	2
χ_4	1	-1	1	1	-1	-1	-i	i	2
χ_5	2	2	c ₂	c ₁	c ₂	c ₁	0	0	3
χ_6	2	2	c ₁	c ₂	c ₁	c ₂	0	0	3
χ_7	2	-2	-c ₂	-c ₁	-c ₂	-c ₁	0	0	0
χ_8	2	-2	-c ₁	-c ₂	-c ₁	-c ₂	0	0	0
\wedge	1	2	5	5	10	10	4	4	

$$c_1 = 2\cos(2\pi/5) = \frac{-1 + \sqrt{5}}{2}, \quad c_2 = 2\cos(4\pi/5) = \frac{-1 - \sqrt{5}}{2}$$

Normal subgroups

Classes	H	G/H	\cap	
1	1 + 2 + 3 + 4 + 5 + 6	10	2	1 Sylow subgroups: [4] × 5, [5]
2	1 + 3 + 4	5	4	2 G' Maximal subgroups: [4], [10]
3	1 + 2	2	D ₁₀	3 Z $\mathfrak{K} = \Phi = [1]$

$$20.5 = M_{5,4}^{(3)} = \langle A^5, B^4, BA = A^3B \rangle$$

CC	1	2	3	4	5			
elts	1	A	A ²	A ³	A ⁴	A ⁿ B ²	A ⁿ B	A ⁿ B ³

C	1	2	3	4	5	
#	1	4	5	5	5	\mathfrak{K}
χ_1	1	1	1	1	1	G
χ_2	1	1	1	-1	-1	1
χ_3	1	1	-1	i	-i	2
χ_4	1	1	-1	-i	i	2
χ_5	4	-1	0	0	0	0
\wedge	1	5	2	4	4	

Normal subgroups

Classes	H	G/H	\cap	
1	1 + 2 + 3	D ₁₀	2	1 Sylow subgroups: [4] × 5, [5]
2	1 + 2	5	4	2 G' Maximal subgroups: [D ₁₀], [4] × 5

$$21 = \langle A^{21} \rangle = 7 \times 3$$

$$21.2 = M_{7,3}^{(4)} = \langle A^7, B^3, BA = A^4B \rangle$$

CC	1	2	3	4	5
elts	1	A A ² A ⁴	A ³ A ⁵ A ⁶	A ⁿ B	A ⁿ B ²

C	1	2	3	4	5	
#	1	3	3	7	7	\mathfrak{K}
χ_1	1	1	1	1	1	G
χ_2	1	1	1	ω	ω^2	1
χ_3	1	1	1	ω^2	ω	1
χ_4	3	a	b	0	0	0
χ_5	3	b	a	0	0	0
\wedge	1	7	7	3	3	

$$a = e^{2\pi/7} + e^{4\pi/7} + e^{8\pi/7} = \frac{-1 + \sqrt{7}i}{2}, \quad b = e^{6\pi/7} + e^{10\pi/7} + e^{12\pi/7} = \frac{-1 - \sqrt{7}i}{2}$$

Normal subgroups

Classes	H	G/H	Sylow and maximal subgroups: [3] × 7, [7]
1	1 + 2 + 3	7	3

$G' \quad Z = \mathfrak{K} = \Phi = [1]$

$$22 = \langle A^{11}, B^2 \rangle = 11 \times 2$$

$$22.2 = D_{22} = \langle A^{11}, B^2, BA = A^{-1}B \rangle$$

CC	1	2	3	4	5	6	7
elts	1	A A ¹⁰	A ² A ⁹	A ³ A ⁸	A ⁴ A ⁷	A ⁵ A ⁶	A ⁿ B

C	1	2	3	4	5	6	7	
#	1	2	2	2	2	2	11	\mathfrak{K}
χ_1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	-1	1
χ_3	2	c ₁	c ₂	c ₃	c ₄	c ₅	0	0
χ_4	2	c ₂	c ₄	c ₅	c ₃	c ₁	0	0
χ_5	2	c ₃	c ₅	c ₂	c ₁	c ₄	0	0
χ_6	2	c ₄	c ₃	c ₁	c ₅	c ₂	0	0
χ_7	2	c ₅	c ₁	c ₄	c ₂	c ₃	0	0
\wedge	1	11	11	11	11	11	2	

$$c_k = 2\cos(2k\pi/11)$$

Normal subgroups

Classes	H	G/H	Sylow and maximal subgroups: [2] × 11, [11]
1	1 + 2 + 3 + 4 + 5 + 6	11	2

$G' \quad Z = \mathfrak{K} = \Phi = [1]$

$$23 = \langle A^{23} \rangle$$

$$24 = \langle A^8, B^3 \rangle = 8 \times 3$$

$$24.02 = \langle A^{12}, B^2 \rangle = 12 \times 2$$

$$24.03 = \langle A^6, B^2, C^2 \rangle = 6 \times 2^2$$

$$24.04 = D_8 \times 3 = \langle A^4, B^2, C^3, BA = A^{-1}B \rangle$$

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
#	1	1	2	2	2	1	1	2	2	2	1	1	2	2	2	\mathfrak{X}
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1	1
χ_3	1	1	-1	1	-1	1	1	-1	1	-1	1	1	-1	1	-1	2
χ_4	1	1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1	1	3
χ_5	2	-2	0	0	0	2	-2	0	0	0	2	-2	0	0	0	4
χ_6	1	1	1	1	1	ω	ω	ω	ω	ω	ω^2	ω^2	ω^2	ω^2	ω^2	5
χ_7	1	1	1	-1	-1	ω	ω	ω	$-\omega$	$-\omega$	ω^2	ω^2	ω^2	$-\omega^2$	$-\omega^2$	6
χ_8	1	1	-1	1	-1	ω	ω	$-\omega$	ω	$-\omega$	ω^2	ω^2	$-\omega^2$	ω^2	$-\omega^2$	7
χ_9	1	1	-1	-1	1	ω	ω	$-\omega$	$-\omega$	ω	ω^2	ω^2	$-\omega^2$	$-\omega^2$	ω^2	8
χ_{10}	2	-2	0	0	0	2ω	-2ω	0	0	0	$2\omega^2$	$-2\omega^2$	0	0	0	0
χ_{11}	1	1	1	1	1	ω^2	ω^2	ω^2	ω^2	ω^2	ω	ω	ω	ω	ω	5
χ_{12}	1	1	1	-1	-1	ω^2	ω^2	ω^2	$-\omega^2$	$-\omega^2$	ω	ω	ω	$-\omega$	$-\omega$	6
χ_{13}	1	1	-1	1	-1	ω^2	ω^2	$-\omega^2$	ω^2	$-\omega^2$	ω	ω	$-\omega$	ω	$-\omega$	7
χ_{14}	1	1	-1	-1	1	ω^2	ω^2	$-\omega^2$	$-\omega^2$	ω^2	ω	ω	$-\omega$	$-\omega$	ω	8
χ_{15}	2	-2	0	0	0	$2\omega^2$	$-2\omega^2$	0	0	0	2ω	-2ω	0	0	0	0
\wedge	1	2	4	2	2	3	6	12	6	6	3	6	12	6	6	

Normal subgroups

Classes	H	G/H	\cap	
1	1 + 2 + 3 + 6 + 7 + 8 + 11 + 12 + 13	12	2	1
2	1 + 2 + 4 + 6 + 7 + 9 + 11 + 12 + 14	6×2	2	2
3	1 + 2 + 5 + 6 + 7 + 10 + 11 + 12 + 15	6×2	2	3
4	1 + 6 + 11	3	D_8	4
5	1 + 2 + 3 + 4 + 5	D_8	3	5
6	1 + 2 + 3	4	6	6
7	1 + 2 + 4	2^2	6	7
8	1 + 2 + 5	2^2	6	8
9	1 + 2 + 6 + 7 + 11 + 12	6	4	$1 \cap 2 \quad \mathbf{Z}$
10	1 + 2	2	6×2	$6 \cap 7 \quad \mathbf{G}'$

$$\mathfrak{X} = \Phi = [1]$$

Sylow subgroups: $[D_8], [3]$ Maximal subgroups: $[6 \times 2] \times 2, [D_8]$

$$24.05 = Q_8 \times 3 = \langle A^4, B^2 = A^2, C^3, BA = A^{-1}B \rangle$$

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
#	1	1	2	2	2	1	1	2	2	2	1	1	2	2	2	\mathfrak{K}
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1	1
χ_3	1	1	-1	1	-1	1	1	-1	1	-1	1	1	-1	1	-1	2
χ_4	1	1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1	1	3
χ_5	2	-2	0	0	0	2	-2	0	0	0	2	-2	0	0	0	4
χ_6	1	1	1	1	1	ω	ω	ω	ω	ω	ω^2	ω^2	ω^2	ω^2	ω^2	5
χ_7	1	1	1	-1	-1	ω	ω	ω	$-\omega$	$-\omega$	ω^2	ω^2	ω^2	$-\omega^2$	$-\omega^2$	6
χ_8	1	1	-1	1	-1	ω	ω	$-\omega$	ω	$-\omega$	ω^2	ω^2	$-\omega^2$	ω^2	$-\omega^2$	7
χ_9	1	1	-1	-1	1	ω	ω	$-\omega$	$-\omega$	ω	ω^2	ω^2	$-\omega^2$	$-\omega^2$	ω^2	8
χ_{10}	2	-2	0	0	0	2ω	-2ω	0	0	0	$2\omega^2$	$-2\omega^2$	0	0	0	0
χ_{11}	1	1	1	1	1	ω^2	ω^2	ω^2	ω^2	ω^2	ω	ω	ω	ω	ω	5
χ_{12}	1	1	1	-1	-1	ω^2	ω^2	ω^2	$-\omega^2$	$-\omega^2$	ω	ω	ω	$-\omega$	$-\omega$	6
χ_{13}	1	1	-1	1	-1	ω^2	ω^2	$-\omega^2$	ω^2	$-\omega^2$	ω	ω	$-\omega$	ω	$-\omega$	7
χ_{14}	1	1	-1	-1	1	ω^2	ω^2	$-\omega^2$	$-\omega^2$	ω^2	ω	ω	$-\omega$	$-\omega$	ω	8
χ_{15}	2	-2	0	0	0	$2\omega^2$	$-2\omega^2$	0	0	0	2ω	-2ω	0	0	0	0
\wedge	1	2	4	4	4	3	6	12	12	12	3	6	12	12	12	

Normal subgroups

Classes	H	G/H	\cap	
1	1 + 2 + 3 + 6 + 7 + 8 + 11 + 12 + 13	12	2	1
2	1 + 2 + 4 + 6 + 7 + 9 + 11 + 12 + 14	12	2	2
3	1 + 2 + 5 + 6 + 7 + 10 + 11 + 12 + 15	12	2	3
4	1 + 6 + 11	3	Q_8	4
5	1 + 2 + 3 + 4 + 5	Q_8	3	5
6	1 + 2 + 3	4	6	6
7	1 + 2 + 4	4	6	7
8	1 + 2 + 5	4	6	8
9	1 + 2 + 6 + 7 + 11 + 12	6	2^2	$1 \cap 2 \quad Z$
10	1 + 2	2	$2^2 \times 3$	$6 \cap 7 \quad G'$

$$N = \Phi = [1]$$

Sylow subgroups: $[Q_8], [3]$ Maximal subgroups: $[Q_8], [12] \times 3$

$$24.06 = D_6 \times 2^2 = \langle A^3, B^2, C^2, D^2, BA = A^{-1}B \rangle$$

	C	1	2	3	4	5	6	7	8	9	10	11	12	
	#	1	2	3	1	2	3	1	2	3	1	2	3	\mathcal{K}
χ_1		1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2		1	1	-1	1	1	-1	1	1	-1	1	1	-1	1
χ_3		2	-1	0	2	-1	0	2	-1	0	2	-1	0	2
χ_4		1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	3
χ_5		1	1	-1	1	1	-1	-1	-1	1	-1	-1	1	4
χ_6		2	-1	0	2	-1	0	-2	1	0	-2	1	0	5
χ_7		1	1	1	-1	-1	-1	1	1	1	-1	-1	-1	6
χ_8		1	1	-1	-1	-1	1	1	1	-1	-1	-1	1	7
χ_9		2	-1	0	-2	1	0	2	-1	0	-2	1	0	8
χ_{10}		1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	9
χ_{11}		1	1	-1	-1	-1	1	-1	-1	1	1	1	-1	10
χ_{12}		2	-1	0	-2	1	0	-2	1	0	2	-1	0	11
\wedge		1	3	2	2	6	2	2	6	2	2	6	2	

Normal subgroups

	Classes	H	G/H	\mathcal{K}	
1	1 + 2 + 4 + 5 + 7 + 8 + 10 + 11	6×2	2	1	
2	1 + 4 + 7 + 10	2^2	D_6	2	$Z = \mathcal{K}$
3	1 + 2 + 3 + 4 + 5 + 6	D_{12}	2	3	
4	1 + 2 + 4 + 5 + 9 + 12	D_{12}	2	4	
5	1 + 4	2	$D_6 \times 2$	5	
6	1 + 2 + 3 + 7 + 8 + 9	D_{12}	2	6	
7	1 + 2 + 6 + 7 + 8 + 12	D_{12}	2	7	
8	1 + 7	2	$D_6 \times 2$	8	
9	1 + 2 + 3 + 10 + 11 + 12	D_{12}	2	9	
10	1 + 2 + 6 + 9 + 10 + 11	D_{12}	2	10	
11	1 + 10	2	$D_6 \times 2$	11	
12	1 + 2 + 4 + 5	6	2^2	1 \cap 3	
13	1 + 2 + 7 + 8	6	2^2	1 \cap 6	
14	1 + 2 + 10 + 11	6	2^2	1 \cap 9	
15	1 + 2 + 3	D_6	2^2	3 \cap 6	
16	1 + 2 + 6	D_6	2^2	3 \cap 7	
17	1 + 2 + 9	D_6	2^2	4 \cap 6	
18	1 + 2 + 12	D_6	2^2	4 \cap 7	
19	1 + 2	3	2^3	12 \cap 13	G'

$$\Phi = [1]$$

Sylow subgroups: $[2^3] \times 3, [3]$ Maximal subgroups: $[2^3] \times 3, [D_6 \times 2] \times 6$

$$24.07 = D_6 \times 4 = \langle A^3, B^2, C^4, BA = A^{-1}B \rangle$$

C	1	2	3	4	5	6	7	8	9	10	11	12	
#	1	2	3	1	2	3	1	2	3	1	2	3	\mathfrak{K}
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	-1	1	1	-1	1	1	-1	1	1	-1	1
χ_3	2	-1	0	2	-1	0	2	-1	0	2	-1	0	2
χ_4	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	3
χ_5	1	1	-1	1	1	-1	-1	-1	1	-1	-1	1	4
χ_6	2	-1	0	2	-1	0	-2	1	0	-2	1	0	5
χ_7	1	1	1	-1	-1	-1	i	i	i	-i	-i	-i	6
χ_8	1	1	-1	-1	-1	1	i	i	-i	-i	-i	i	7
χ_9	2	-1	0	-2	1	0	2i	-i	0	-2i	i	0	0
χ_{10}	1	1	1	-1	-1	-1	-i	-i	-i	i	i	i	6
χ_{11}	1	1	-1	-1	-1	1	-i	-i	i	i	i	-i	7
χ_{12}	2	-1	0	-2	1	0	-2i	i	0	2i	-i	0	0
\wedge	1	3	2	2	6	2	4	12	4	4	12	4	

Normal subgroups

Classes	H	G/H	\cap	
1 1+ 2+ 4+ 5+ 7+ 8+ 10+ 11	12	2	1	
2 1+ 4+ 7+ 10	4	D_6	2	$Z = Z_2 = \mathfrak{K}$
3 1+ 2+ 3+ 4+ 5+ 6	D_{12}	2	3	
4 1+ 2+ 4+ 5+ 9+ 12	$D_{3,4}$	2	4	
5 1+ 4	2	$D_6 \times 2$	5	Φ
6 1+ 2+ 3	D_6	4	6	
7 1+ 2+ 6	D_6	4	7	
8 1+ 2+ 4+ 5	6	2^2	$1 \cap 3$	
9 1+ 2	3	4×2	$1 \cap 7$	G'

Sylow subgroups: $[4 \times 2] \times 3$, $[3]$ Maximal subgroups: $[12]$, $[D_6 \times 2]$, $[D_{3,4}]$, $[4 \times 2] \times 3$



$$24.08 = 12.4 \times 2 = \langle A^3, B^4, C^2, BA = A^{-1}B \rangle$$

	C	1	2	3	4	5	6	7	8	9	10	11	12	
#	1	1	2	2	3	3	1	1	2	2	3	3		\mathcal{K}
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	-1	-1	1	1	1	1	-1	-1	1	
χ_3	1	-1	1	-1	i	-i	1	-1	1	-1	i	-i	2	
χ_4	1	-1	1	-1	-i	i	1	-1	1	-1	-i	i	2	
χ_5	2	2	-1	-1	0	0	2	2	-1	-1	0	0	3	
χ_6	2	-2	-1	1	0	0	2	-2	-1	1	0	0	4	
χ_7	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	5	
χ_8	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	6	
χ_9	1	-1	1	-1	i	-i	-1	1	-1	1	-i	i	7	
χ_{10}	1	-1	1	-1	-i	i	-1	1	-1	1	i	-i	7	
χ_{11}	2	2	-1	-1	0	0	-2	-2	1	1	0	0	8	
χ_{12}	2	-2	-1	1	0	0	-2	2	1	-1	0	0	9	
\wedge	1	2	3	6	4	4	2	2	6	6	4	4		

Normal subgroups

	Classes	H	G/H	\cap	
1	1 + 2 + 3 + 4 + 7 + 8 + 9 + 10	6×2	2	1	
2	1 + 3 + 7 + 9	6	4	2	
3	1 + 2 + 7 + 8	2^2	D₆	3	$Z = \mathcal{Z}$
4	1 + 7	2	12.4	4	
5	1 + 2 + 3 + 4 + 5 + 6	D_{3,4}	2	5	
6	1 + 2 + 3 + 4 + 11 + 12	D_{3,4}	2	6	
7	1 + 3 + 8 + 10	6	4	7	
8	1 + 2	2	D₆ × 2	8	Φ
9	1 + 8	2	12.4	9	
10	1 + 2 + 3 + 4	6	2^2	$1 \cap 5$	
11	1 + 3	3	4×2	$2 \cap 5$	G'

Sylow subgroups: $[4 \times 2] \times 3$, $[3]$ Maximal subgroups: $[6 \times 2]$, $[D_{3,4}] \times 2$, $[4 \times 2] \times 3$

$$24.09 = D_{3,8} = \langle A^3, B^8, BA = A^{-1}B \rangle$$

CC	1	2	3	4	5	6	7	8	9	10	11	12
elts	1	B ⁴	B ²	B ⁶	A	AB ⁴	AB ²	AB ⁶	A ⁿ B	A ⁿ B ⁷	A ⁿ B ³	A ⁿ B ⁵
					A ²	A ² B ⁴	A ² B ²	A ² B ⁶				

C	1	2	3	4	5	6	7	8	9	10	11	12	
#	1	1	1	1	2	2	2	2	3	3	3	3	\mathcal{K}
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	-1	-1	-1	-1	1
χ_3	1	1	-1	-1	1	1	-1	-1	i	-i	i	-i	2
χ_4	1	1	-1	-1	1	1	-1	-1	-i	i	-i	i	2
χ_5	1	-1	i	-i	1	-1	i	-i	θ	-i θ	- θ	i θ	3
χ_6	1	-1	-i	i	1	-1	-i	i	-i θ	θ	i θ	- θ	3
χ_7	1	-1	i	-i	1	-1	i	-i	- θ	i θ	θ	-i θ	3
χ_8	1	-1	-i	i	1	-1	-i	i	i θ	- θ	-i θ	θ	3
χ_9	2	2	2	2	-1	-1	-1	-1	0	0	0	0	4
χ_{10}	2	2	-2	-2	-1	-1	1	1	0	0	0	0	5
χ_{11}	2	-2	2i	-2i	-1	1	-i	i	0	0	0	0	0
χ_{12}	2	-2	-2i	2i	-1	1	i	-i	0	0	0	0	0
order	1	2	4	4	3	6	12	12	8	8	8	8	

$$\theta = e^{2\pi/8} = \frac{1+i}{\sqrt{2}}$$

Normal subgroups

	Classes	H	G/H	\cap
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8	12	2	1
2	1 + 2 + 5 + 6	6	4	2
3	1 + 5	3	8	3 G'
4	1 + 2 + 3 + 4	4	D_6	4 $Z = \Phi = \mathcal{K}$
5	1 + 2	2	12.4	5

Sylow subgroups: [8] \times 3, [3] Maximal subgroups: [12], [8] \times 3

$$24.10 = A_4 \times 2 = \langle A^2, B^2, C^3, D^2, CA = ABC, CB = AC \rangle$$

	C	1	2	3	4	5	6	7	8	
#		1	3	4	4	1	3	4	4	\mathcal{K}
χ_1		1	1	1	1	1	1	1	1	G
χ_2		1	1	ω	ω^2	1	1	ω	ω^2	1
χ_3		1	1	ω^2	ω	1	1	ω^2	ω	1
χ_4		3	-1	0	0	3	-1	0	0	2
χ_5		1	1	1	1	-1	-1	-1	-1	3
χ_6		1	1	ω	ω^2	-1	-1	$-\omega$	$-\omega^2$	4
χ_7		1	1	ω^2	ω	-1	-1	$-\omega^2$	$-\omega$	4
χ_8		3	-1	0	0	-3	1	0	0	0
\wedge		1	2	3	3	2	2	6	6	

Normal subgroups

Classes	H	G/H	\cap
1	1 + 2 + 5 + 6	2^3	3
2	1 + 5	2	A_4
3	1 + 2 + 3 + 4	A_4	2
4	1 + 2	2^2	6

$Z = \mathcal{Z}$
 G'

Sylow subgroups: $[2^3], [3] \times 4$ Maximal subgroups: $[A_4], [2^3], [6] \times 4$



$$24.11 = \langle A^3, B^4, C^2, BA = A^{-1}B, CB = B^{-1}C \rangle$$

CC	1	2	3	4	5	6	7	8	9
elts	1	B ²	A A ²	AB ² A ² B ²	C B ² C	AC A ² B ² C	A ² C A ² B ² C	A ⁿ BC A ⁿ B ³ C	A ⁿ B A ⁿ B ³

C	1	2	3	4	5	6	7	8	9	
#	1	1	2	2	2	2	2	6	6	\mathcal{K}
χ_1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	-1	-1	1
χ_3	1	1	1	1	-1	-1	-1	1	-1	2
χ_4	1	1	1	1	-1	-1	-1	-1	1	3
χ_5	2	2	-1	-1	2	-1	-1	0	0	4
χ_6	2	2	-1	-1	-2	1	1	0	0	5
χ_7	2	-2	2	-2	0	0	0	0	0	6
χ_8	2	-2	-1	1	0	$\sqrt{3}i$	$-\sqrt{3}i$	0	0	0
χ_9	2	-2	-1	1	0	$-\sqrt{3}i$	$\sqrt{3}i$	0	0	0
\wedge	1	2	3	6	2	6	6	2	4	

Normal subgroups

	Classes	H	G/H	\cap	
1	1+ 2+ 3+ 4+ 5+ 6+ 7	6×2	2	1	
2	1+ 2+ 3+ 4+ 8	D_{12}	2	2	
3	1+ 2+ 3+ 4+ 9	$D_{3,4}$	2	3	
4	1+ 2+ 5	2^2	D_6	4	
5	1+ 2	2		5	Z
6	1+ 3	3	2^3	6	G'
7	1+ 2+ 3+ 4	6	2^2	1 \cap 2	

$$\mathcal{O} = \Phi = [1]$$

Sylow subgroups: $[D_8] \times 3$, $[3]$ Maximal subgroups: $[6 \times 2]$, $[D_6 \times 2]$, $[D_{3,4}]$, $[D_8] \times 3$



$$24.12 = D_{24} = \langle A^{12}, B^2, BA = A^{-1}B \rangle$$

CC	1	2	3	4	5	6	7	8	9
elts	1	A^6	A^3	A^4	A^2	A	A^5	$A^{2n}B$	$A^{2n+1}B$
			A^9	A^8	A^{10}	A^{11}	A^7		

C	1	2	3	4	5	6	7	8	9	
#	1	1	2	2	2	2	2	6	6	\mathfrak{K}
χ_1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	-1	-1	1
χ_3	1	1	-1	1	1	-1	-1	1	-1	2
χ_4	1	1	-1	1	1	-1	-1	-1	1	3
χ_5	2	2	2	-1	-1	-1	-1	0	0	4
χ_6	2	2	-2	-1	-1	1	1	0	0	5
χ_7	2	-2	0	2	-2	0	0	0	0	6
χ_8	2	-2	0	-1	1	$\sqrt{3}$	$-\sqrt{3}$	0	0	0
χ_9	2	-2	0	-1	1	$-\sqrt{3}$	$\sqrt{3}$	0	0	0
\wedge	1	2	4	3	6	12	12	2	2	

Normal subgroups

	Classes	H	G/H	\cap	
1	1 + 2 + 3 + 4 + 5 + 6 + 7	12	2	1	
2	1 + 2 + 4 + 5 + 8	D_{12}	2	2	
3	1 + 2 + 4 + 5 + 9	D_{12}	2	3	
4	1 + 2 + 3	4	D_6	4	
5	1 + 2	2	D_{12}	5	$Z = \Phi = \mathfrak{K}$
6	1 + 4	3	D_8	6	
7	1 + 2 + 4 + 5	6	2^2	$1 \cap 2$	G'

Sylow subgroups: $[D_8] \times 3$, $[3]$ Maximal subgroups: $[12]$, $[D_6 \times 2] \times 2$, $[D_8] \times 3$



$$24.13 = Q_{24} = \langle A^{12}, B^2 = A^6, BA = A^{-1}B \rangle$$

CC	1	2	3	4	5	6	7	8	9
elts	1	A^6	A^3	A^4	A^2	A	A^5	$A^{2n}B$	$A^{2n+1}B$
			A^9	A^8	A^{10}	A^{11}	A^7		

C	1	2	3	4	5	6	7	8	9	
#	1	1	2	2	2	2	2	6	6	\mathfrak{K}
χ_1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	-1	-1	1
χ_3	1	1	-1	1	1	-1	-1	1	-1	2
χ_4	1	1	-1	1	1	-1	-1	-1	1	3
χ_5	2	2	2	-1	-1	-1	-1	0	0	4
χ_6	2	2	-2	-1	-1	1	1	0	0	5
χ_7	2	-2	0	2	-2	0	0	0	0	6
χ_8	2	-2	0	-1	1	$\sqrt{3}$	$-\sqrt{3}$	0	0	0
χ_9	2	-2	0	-1	1	$-\sqrt{3}$	$\sqrt{3}$	0	0	0
\wedge	1	2	4	3	6	12	12	4	4	

Normal subgroups

Classes	H	G/H	\cap	
1	1 + 2 + 3 + 4 + 5 + 6 + 7	12	2	1
2	1 + 2 + 4 + 5 + 8	$D_{3,4}$	2	2
3	1 + 2 + 4 + 5 + 9	$D_{3,4}$	2	3
4	1 + 2 + 3	4	D_6	4 $Z_2 = \mathfrak{K}$
5	1 + 2	2	D_{12}	5 $Z = \Phi$
6	1 + 4	3	Q_8	6
7	1 + 2 + 4 + 5	6	2^2	1 \cap 2 G'

Sylow subgroups: $[Q_8] \times 3$, $[3]$ Maximal subgroups: $[12]$, $[D_{3,4}] \times 2$, $[Q_8] \times 3$



24.14 = SL(2, 3)

$= \langle A^4, B^2 = A^2, C^3, BA = A^{-1}B, CA = A^{-1}BC, CB = A^{-1}C \rangle$

CC	1	2	3	4	5	6	7
elts	1	A ²	A B A ³ AB A ² B A ³ B	C AC A ² BC A ³ BC	C ² BC ² ABC ² A ³ C ²	AC ² A ² C ² A ² BC ² A ³ BC ²	A ² C A ³ C BC ABC

C	1	2	3	4	5	6	7	
#	1	1	6	4	4	4	4	\mathfrak{K}
χ_1	1	1	1	1	1	1	1	G
χ_2	1	1	1	ω	ω^2	ω^2	ω	1
χ_3	1	1	1	ω^2	ω	ω	ω^2	1
χ_4	2	-2	0	-1	-1	1	1	0
χ_5	2	-2	0	$-\omega$	$-\omega^2$	ω^2	ω	0
χ_6	2	-2	0	$-\omega^2$	$-\omega$	ω	ω^2	0
χ_7	3	3	-1	0	0	0	0	2
^	1	2	4	3	3	6	6	

Normal subgroups

Classes	H	G/H	\cap	
1	1+2+3	Q ₈	3	1 G'
2	1+2	2	A ₄	2 Z = Φ = ℳ

Sylow subgroups: [Q₈], [3 × 4]
Maximal subgroups: [Q₈], [6] × 4

24.15 = S₄

CC	1	2	3	4	5
elts	1	(xx)(xx)	(xx)	(xxxx)	(xxx)

C	1	2	3	4	5	
#	1	3	6	6	8	\mathfrak{K}
χ_1	1	1	1	1	1	G
χ_2	1	1	-1	-1	1	1
χ_3	2	2	0	0	-1	2
χ_4	3	-1	1	-1	0	0
χ_5	3	-1	-1	1	0	0
^	1	2	2	4	3	

Normal subgroups

Classes	H	G/H	\cap	Z = ℳ = Φ = [1]
1	1+2+5	A ₄	2	1 G'
2	1+2	2 ²	D ₆	2

Sylow subgroups: [Q₈] × 3, [3] × 4
Maximal subgroups: [A₄], [D₈] × 3

$$25 = \langle A^{25} \rangle$$

$$25.2 = 5^2 = \langle A^5, B^5 \rangle$$

$$26 = \langle A^{26} \rangle = 13 \times 2$$

$$26.2 = \langle A^{13}, B^2, BA = A^{-1}B \rangle = D_{26}$$

CC	1	2	3	4	5	6	7	8
elts	1	A	A ²	A ³	A ⁴	A ⁵	A ⁶	A ⁿ B
		A ¹²	A ¹¹	A ¹⁰	A ⁹	A ⁸	A ⁷	

C	1	2	3	4	5	6	7	8	
#	1	2	2	2	2	2	2	13	\mathfrak{K}
χ_1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	-1	1
χ_3	2	c ₁	c ₂	c ₃	c ₄	c ₅	c ₆	0	0
χ_4	2	c ₂	c ₄	c ₆	c ₅	c ₃	c ₁	0	0
χ_5	2	c ₃	c ₆	c ₄	c ₁	c ₂	c ₅	0	0
χ_6	2	c ₄	c ₅	c ₁	c ₃	c ₆	c ₂	0	0
χ_7	2	c ₅	c ₃	c ₂	c ₆	c ₁	c ₄	0	0
χ_8	2	c ₆	c ₁	c ₅	c ₂	c ₄	c ₃	0	0
\wedge	1	13	13	13	13	13	13	2	

$c_k = 2\cos(2k\pi/13)$

Normal subgroups

Classes	H	G/H	\cap
1 1 + 2 + 3 + 4 + 5 + 6 + 7	13	2	1 G'

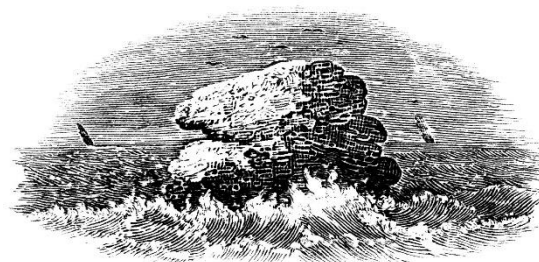
$$Z = N = \Phi = [1]$$

Sylow and maximal subgroups: $[2] \times 13, [13]$

$$27 = \langle A^{27} \rangle$$

$$27.2 = 9 \times 3 = \langle A^9, B^3 \rangle$$

$$27.3 = 3^3 = \langle A^3, B^3, C^3 \rangle$$



$$27.4 = \langle A^3, B^3, C^3, CB = A^2BC \rangle$$

CC	1	2	3	4	5	6	7	8	9	10	11
elts	1	A	A ²	A ⁿ B	A ⁿ B ²	A ⁿ C	A ⁿ C ²	A ⁿ BC	A ⁿ B ² C ²	A ⁿ B ² C	A ⁿ BC ²

C	1	2	3	4	5	6	7	8	9	10	11	
#	1	1	1	3	3	3	3	3	3	3	3	\mathfrak{K}
χ_1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	ω	ω^2	ω^2	ω	ω	ω^2	1
χ_3	1	1	1	1	1	ω^2	ω	ω	ω^2	ω^2	ω	1
χ_4	1	1	1	ω	ω^2	1	1	ω^2	ω	ω^2	ω	2
χ_5	1	1	1	ω^2	ω	1	1	ω	ω^2	ω	ω^2	2
χ_6	1	1	1	ω^2	ω	ω	ω^2	1	1	ω	ω^2	3
χ_7	1	1	1	ω	ω^2	ω^2	ω	1	1	ω	ω^2	3
χ_8	1	1	1	ω	ω^2	ω	ω^2	ω	ω^2	1	1	4
χ_9	1	1	1	ω^2	ω	ω^2	ω	ω^2	ω	1	1	4
χ_{10}	3	3 ω	3 ω^2	0	0	0	0	0	0	0	0	0
χ_{11}	3	3 ω^2	3 ω	0	0	0	0	0	0	0	0	0
\wedge	1	3	3	3	3	3	3	3	3	3	3	

Normal subgroups

	Classes	H	G/H	\cap
1	1 + 2 + 3 + 4 + 5	3^2	3	1
2	1 + 2 + 3 + 6 + 7	3^2	3	2
3	1 + 2 + 3 + 8 + 9	3^2	3	3
4	1 + 2 + 3 + 10 + 11	3^2	3	4
5	1 + 2 + 3	3	3×3	$1 \cap 2$ $G' = Z = \Phi = \mathfrak{K}$

Maximal subgroups: $[3^2] \times 4$



$$27.5 = M_{9,3}^{(4)} = \langle A^9, B^3, BA = A^4B \rangle$$

CC	1	2	3	4	5	6	7	8	9	10	11
elts	1	A^3	A^6	$A^{3n}B$	$A^{3n}B^2$	A^{3n+1}	$A^{3n+2}A^2$	$A^{3n+1}B$	$A^{3n+2}B^2$	$A^{3n+1}B^2$	$A^{3n+2}B$

C	1	2	3	4	5	6	7	8	9	10	11	
#	1	1	1	3	3	3	3	3	3	3	3	\mathfrak{K}
χ_1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	ω	ω^2	ω^2	ω	ω	ω^2	1
χ_3	1	1	1	1	1	ω^2	ω	ω	ω^2	ω^2	ω	1
χ_4	1	1	1	ω	ω^2	1	1	ω^2	ω	ω^2	ω	2
χ_5	1	1	1	ω^2	ω	1	1	ω	ω^2	ω	ω^2	2
χ_6	1	1	1	ω^2	ω	ω	ω^2	1	1	ω	ω^2	3
χ_7	1	1	1	ω	ω^2	ω^2	ω	1	1	ω	ω^2	3
χ_8	1	1	1	ω	ω^2	ω	ω^2	ω	ω^2	1	1	4
χ_9	1	1	1	ω^2	ω	ω^2	ω	ω^2	ω	1	1	4
χ_{10}	3	3ω	$3\omega^2$	0	0	0	0	0	0	0	0	0
χ_{11}	3	$3\omega^2$	3ω	0	0	0	0	0	0	0	0	0
^	1	3	3	3	3	9	9	9	9	9	9	

Normal subgroups

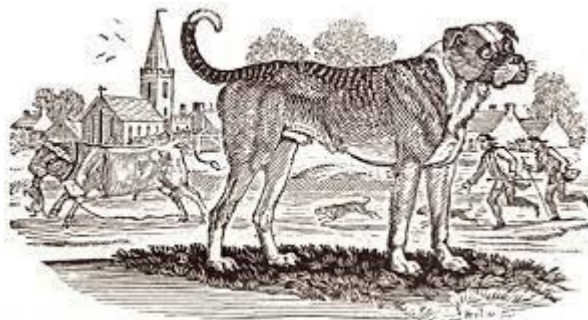
Classes	H	G/H	\cap
1	1+2+3+4+5	3^2	3
2	1+2+3+6+7	9	3
3	1+2+3+8+9	9	3
4	1+2+3+10+11	9	3
5	1+2+3	3	3^2

$1 \cap 2 \quad G' = Z = \Phi$

Maximal subgroups: $[3^2], [9] \times 3$

$$28 = \langle A^{28} \rangle$$

$$28.2 = \langle A^{14}, B^2 \rangle = 14 \times 2$$



$$28.3 = D_{28} = \langle A^7, B^2, C^2, BA = A^{-1}B \rangle = D_{14} \times 2$$

C	1	2	3	4	5	6	7	8	9	10	
#	1	2	2	2	7	1	2	2	2	7	\mathfrak{K}
χ_1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	-1	1	1	1	1	-1	1
χ_3	2	c_1	c_2	c_3	0	2	c_1	c_2	c_3	0	2
χ_4	2	c_3	c_1	c_2	0	2	c_3	c_1	c_2	0	2
χ_5	2	c_2	c_3	c_1	0	2	c_2	c_3	c_1	0	2
χ_6	1	1	1	1	1	-1	-1	-1	-1	-1	3
χ_7	1	1	1	1	-1	-1	-1	-1	-1	1	4
χ_8	2	c_1	c_2	c_3	0	-2	$-c_1$	$-c_2$	$-c_3$	0	0
χ_9	2	c_3	c_1	c_2	0	-2	$-c_3$	$-c_1$	$-c_2$	0	0
χ_{10}	2	c_2	c_3	c_1	0	-2	$-c_2$	$-c_3$	$-c_1$	0	0
\wedge	1	7	7	7	2	2	14	14	14	2	

Normal subgroups

Classes	H	G/H	\cap
1	1 + 2 + 3 + 4 + 5 + 7 + 8 + 9	14	2
2	1 + 6	2	D_{14}
3	1 + 2 + 3 + 4 + 5	D_{14}	2
4	1 + 2 + 3 + 4 + 10	D_{14}	2
5	1 + 2 + 3 + 4	7	2^2

1 \cap 3 G'

$$\mathfrak{K} = \Phi = [1]$$

Sylow subgroups: $[2^2] \times 7$, $[7]$ Maximal subgroups: $[D_{14}] \times 2$, $[14]$, $[2^2] \times 7$



$$28.4 = D_{7,4} = \langle A^7, B^4, BA = A^{-1}B \rangle$$

CC	1	2	3	4	5	6	7	8	9	10
elts	1	B ²	A A ⁶	AB ² A ⁶ B ²	A ² A ⁵	A ² B ² A ⁵ B ²	A ³ A ⁴	AB ² A ³ B ²	A ⁿ B	A ⁿ B ³

C	1	2	3	4	5	6	7	8	9	10	
#	1	1	2	2	2	2	2	2	7	7	\mathcal{K}
χ_1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	-1	-1	1
χ_3	1	-1	1	-1	1	-1	1	-1	i	-i	2
χ_4	1	-1	1	-1	1	-1	1	-1	-i	i	2
χ_5	2	2	c ₁	c ₁	c ₂	c ₂	c ₃	c ₃	0	0	3
χ_6	2	2	c ₂	c ₂	c ₃	c ₃	c ₁	c ₁	0	0	3
χ_7	2	2	c ₃	c ₃	c ₁	c ₁	c ₂	c ₂	0	0	3
χ_8	2	-2	c ₁	-c ₁	c ₂	-c ₂	c ₃	-c ₃	0	0	0
χ_9	2	-2	c ₂	-c ₂	c ₃	-c ₃	c ₁	-c ₁	0	0	0
χ_{10}	2	-2	c ₃	-c ₃	c ₁	-c ₁	c ₂	-c ₂	0	0	0
\wedge	1	2	7	14	7	14	7	14	4	4	

$$c_k = 2\cos(2k\pi/7)$$

Normal subgroups

	Classes	H	G/H	\cap	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8	14	2	1	
2	1 + 3 + 5 + 7	7	4	2	G'
5	1 + 2	2	D₁₄	3	Z

$$N = \Phi = [1]$$

Sylow subgroups: [4] \times 7, [7] Maximal subgroups: [14], [4]

$$29 = \langle A^{29} \rangle$$

$$30 = \langle A^{30} \rangle = 15 \times 2$$



$$30.2 = D_6 \times 5 = \langle A^3, B^2, C^5, BA = A^{-1}B \rangle$$

	C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
	#	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	\mathcal{K}
χ_1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2		1	1	-1	1	1	-1	1	1	-1	1	1	-1	1	1	-1	1
χ_3		2	-1	0	2	-1	0	2	-1	0	2	-1	0	2	-1	0	2
χ_4		1	1	1	θ	θ	θ	θ^2	θ^2	θ^2	θ^3	θ^3	θ^3	θ^4	θ^4	θ^4	3
χ_5		1	1	-1	θ	θ	$-\theta$	θ^2	θ^2	$-\theta^2$	θ^3	θ^3	$-\theta^3$	θ^4	θ^4	$-\theta^4$	4
χ_6		2	-1	0	2θ	$-\theta$	0	$2\theta^2$	$-\theta^2$	0	$2\theta^3$	$-\theta^3$	0	$2\theta^4$	$-\theta^4$	0	0
χ_7		1	1	1	θ^2	θ^2	θ^2	θ^4	θ^4	θ^4	θ	θ	θ	θ^3	θ^3	θ^3	3
χ_8		1	1	-1	θ^2	θ^2	$-\theta^2$	θ^4	θ^4	$-\theta^4$	θ	θ	$-\theta$	θ^3	θ^3	$-\theta^3$	4
χ_9		2	-1	0	$2\theta^2$	$-\theta^2$	0	$2\theta^4$	$-\theta^4$	0	2θ	$-\theta$	0	$2\theta^3$	$-\theta^3$	0	0
χ_{10}		1	1	1	θ^3	θ^3	θ^3	θ	θ	θ	θ^4	θ^4	θ^4	θ^2	θ^2	θ^2	3
χ_{11}		1	1	-1	θ^3	θ^3	$-\theta^3$	θ	θ	$-\theta$	θ^4	θ^4	$-\theta^4$	θ^2	θ^2	$-\theta^2$	4
χ_{12}		2	-1	0	$2\theta^3$	$-\theta^3$	0	2θ	$-\theta$	0	$2\theta^4$	$-\theta^4$	0	$2\theta^2$	$-\theta^2$	0	0
χ_{13}		1	1	1	θ^4	θ^4	θ^4	θ^3	θ^3	θ^3	θ^2	θ^2	θ^2	θ	θ	θ	3
χ_{14}		1	1	-1	θ^4	θ^4	$-\theta^4$	θ^3	θ^3	$-\theta^3$	θ^2	θ^2	$-\theta^2$	θ	θ	$-\theta$	4
χ_{15}		2	-1	0	$2\theta^4$	$-\theta^4$	0	$2\theta^3$	$-\theta^3$	0	$2\theta^2$	$-\theta^2$	0	2θ	$-\theta$	0	0
\wedge		1	3	2	5	15	10	5	15	10	5	15	10	5	15	10	

Normal subgroups

	Classes	H	G/H	\cap
1	1 + 2 + 4 + 5 + 7 + 8 + 10 + 11 + 13 + 14	15	2	1
2	1 + 4 + 7 + 10 + 13	5	D_6	2 $Z = \mathcal{Z}$
3	1 + 2 + 3	D_6	5	3
4	1 + 2	3	10	4 G'

$$\Phi = [1]$$

Sylow subgroups: $[2] \times 3$, $[3]$, $[5]$ Maximal subgroups: $[15]$, $[10] \times 3$, $[D_6]$



$$30.3 = D_{10} \times 3 = \langle A^5, B^2, C^3, BA = A^{-1}B \rangle$$

C	1	2	3	4	5	6	7	8	9	10	11	12	
#	1	2	2	5	1	2	2	5	1	2	2	5	\mathcal{K}
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	-1	1	1	1	-1	1	1	1	-1	1
χ_3	2	c_1	c_2	0	2	c_1	c_2	0	2	c_1	c_2	0	2
χ_4	2	c_2	c_1	0	2	c_2	c_1	0	2	c_2	c_1	0	2
χ_5	1	1	1	1	ω	ω	ω	ω	ω^2	ω^2	ω^2	ω^2	3
χ_6	1	1	1	-1	ω	ω	ω	$-\omega$	ω^2	ω^2	ω^2	$-\omega^2$	4
χ_7	2	c_1	c_2	0	2ω	ωc_1	ωc_2	0	$2\omega^2$	$\omega^2 c_1$	$\omega^2 c_2$	0	0
χ_8	2	c_2	c_1	0	2ω	ωc_2	ωc_1	0	$2\omega^2$	$\omega^2 c_2$	$\omega^2 c_1$	0	0
χ_9	1	1	1	1	ω^2	ω^2	ω^2	ω^2	ω	ω	ω	ω	3
χ_{10}	1	1	1	-1	ω^2	ω^2	ω^2	$-\omega^2$	ω	ω	ω	$-\omega$	4
χ_{11}	2	c_1	c_2	0	$2\omega^2$	$\omega^2 c_1$	$\omega^2 c_2$	0	2ω	ωc_1	ωc_2	0	0
χ_{12}	2	c_2	c_1	0	$2\omega^2$	$\omega^2 c_2$	$\omega^2 c_1$	0	2ω	ωc_2	ωc_1	0	0
\wedge	1	5	5	2	3	15	15	6	3	15	15	6	

Normal subgroups

	Classes	H	G/H	\cap
1	1 + 2 + 3 + 5 + 6 + 7 + 9 + 10 + 11	15	2	1
2	1 + 5 + 9	3	D_{10}	2 $Z = \mathcal{K}$
3	1 + 2 + 3 + 4	D_{10}	3	3
4	1 + 2 + 3	5	6	4 G'

$$\Phi = [1]$$

Sylow subgroups: $[2] \times 5$, $[3]$, $[5]$ Maximal subgroups: $[15]$, $[D_{10}]$, $[6] \times 5$



$$30.4 = D_{30} = \langle A^{15}, B^2, BA = A^{-1}B \rangle$$

CC	1	2	3	4	5	6	7	8	9
elts	1	A ⁵ A ¹⁰	A A ¹⁴	A ⁴ A ¹¹	A ⁶ A ⁹	A ² A ¹³	A ³ A ¹²	A ⁷ A ⁸	A ⁿ B

C	1	2	3	4	5	6	7	8	9	
#	1	2	2	2	2	2	2	2	15	\mathcal{K}
χ_1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	1	1	1	1	-1	1
χ_3	2	-1	-1	-1	2	-1	2	-1	0	2
χ_4	2	-1	c ₁	c ₄	c ₆	c ₂	c ₃	c ₇	0	0
χ_5	2	2	c ₂	c ₇	c ₃	c ₄	c ₆	c ₁	0	3
χ_6	2	-1	c ₃	c ₃	c ₃	c ₆	c ₆	c ₆	0	0
χ_7	2	-1	c ₄	c ₁	c ₆	c ₇	c ₃	c ₂	0	0
χ_8	2	2	c ₆	c ₆	c ₆	c ₃	c ₃	c ₃	0	3
χ_9	2	-1	c ₇	c ₂	c ₃	c ₁	c ₆	c ₄	0	0
\wedge	1	3	15	15	5	15	5	15	2	

$c_k = 2\cos(2k\pi/15)$

Normal subgroups

	Classes	H	G/H	\cap	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8	15	2	1	G'
2	1 + 5 + 7	5	D_6	2	
3	1 + 2	3	D_{10}	3	

$$Z = \mathcal{Z} = \Phi = [1]$$

Sylow subgroups: [2] \times 15, [3], [5] Maximal subgroups: [15], [D_{10}] \times 3, [5] \times 5

$$31 = \langle A^{31} \rangle$$

